ANNEXURE A

1 METHOD USED TO DETERMINE ECLIPSE MINIMA

The minimum of a light curve eclipse is determined by performing a second order quadratic curve fit to the data surrounding the anticipated or expected minimum. Therefore if there is an instance of k number of data points in close proximity (close proximity is regarded as not more than 0.06 of phase from the expected eclipse minimum), a polynomial fit is done by making use of second order regression analysis in the form of eq. 1:

$$y = ax^2 + bx + c \tag{1}$$

T_{min} which is the time of the minimum of the eclipse, is determined where the gradient (differentiated with respect to x) of this quadratic equation equals zero, thus:

$$T_{min} = \frac{-b}{2a} \tag{2}$$

Consequently by making use of the variance and covariance definitions, the variance of a function $f(x,y)$ is as follows, equation 3, (Feigelson & Babu, 2012):

$$\mathrm{Var}(f(x,y)) = \mathrm{Var}(x)\left(\left.\frac{\partial f(x,y)}{\partial x}\right|_{x,y=\tilde{x},\tilde{y}}\right)^2 + \mathrm{Var}(y)\left(\left.\frac{\partial f(x,y)}{\partial y}\right|_{x,y=\tilde{x},\tilde{y}}\right)^2 + 2\mathrm{Cov}(x,y)\left(\frac{\partial f(x,y)}{\partial x}\right)\left.\left(\frac{\partial f(x,y)}{\partial y}\right)\right|_{x,y=\tilde{x},\tilde{y}} \tag{3}$$

$Var(f(x,y)$ is the variance of the function $f(x,y)$, $var(x)$, $var(y)$ the variance in x and y respectively. The error at T_{min}, namely $Tmin_{err}$ is given by the square root of:

$$\mathrm{Var}(T_{min}) = \frac{b^2}{4a^4}\mathrm{Var}(a) + \frac{1}{4a^2}\mathrm{Var}(b) - \frac{b}{2a^3}\mathrm{Cov}(a,b) \tag{4}$$

From eq. 4, the error in $Tmin_{err}$ is determined by:

$$Tmin_{err} = (\mathrm{Var}(T_{min}))^{0.5} \tag{5}$$

These values of $Tmin_{err}$ are amongst those quoted in this edition of the Bundesdeutsche Arbeitsgemeinschaft für Veränderliche Sterne e.V., **BAV Journal**.

A Python routine is used to perform the curve fit of each eclipse minimum and thus to derive the values of the parameters a, b and c, as well as the variances, namely Var(a), Var(b) and the covariance, Cov (a,b), in accordance with equation 4.

To demonstrate how these in practice provided the results as reflected in this edition of the **BAV Journal** regarding the W UMa contact binary system DY CET, a few examples (that had 6 data points) were selected.

Included are images taken from Excel that show the results of these minima and that indeed the errors are as determined with small margins:

These data minima were calculated making use of a Python programme where *'wasperr'* refers to the name of this particular Python routine provided and developed by Ms Patricia Skelton, (Smits & Skelton, 2019).

2 Conclusion

In conclusion the method and a recalculation of some of the data points are included to demonstrate that it is possible and indeed accurate to have such small error values for the times of minima as submitted in this edition of the **BAV Journal**. Dependencies may be on the goodness of fit of the data and the type of Python routine performing the fit.

time mag mag_err		
0 0.485840 9.964859 0.012260	Input data	
1 0.486328 9.978423 0.012039		
2 0.500000 10.000257 0.009781		
3 0.500488 10.002438 0.009782		
4 0.515137 9.971275 0.008394		
5 0.515625 9.957190 0.008595		
**************************************		
-157.53292834817609 157.51576487653105 -29.37289598530854	2455803.4999 V	0.0008
T_min1 (waspper) = 0.4999455241776275		
var a1= 783.1185136830428		
var b1= 788.6013895251875		
covar ab1= -785.826452132998		
var c1= 49.58599317724733		
errort_min1 waspper= 0.0008252139624369438		
**************************************		
[-157.53292835 157.51576488 -29.37289599] [[1650.15607265 -1655.86213247 415.17065841]		
[-1655.86213247 1661.70936977 -416.66747096]		
[415.17065841 -416.66747096 104.48562552]]		
**************************************		

Figure 1: Example of Re-calculation 1

time mag mag_err		
0 0.451172 9.952394 0.005727	Input data	
1 0.451660 9.975769 0.006240		
2 0.466309 10.003016 0.005498		
3 0.466797 10.006867 0.005499		
4 0.481445 9.959496 0.005497		
5 0.481934 9.973846 0.005530		
**************************************		
-174.85260431344398 163.28018783545588 -28.113435880077464	2455874.4668 V	0.0011
T_min1 (waspper) = 0.46690808088496305		
var a1= 1610.8575881899114		
var b1= 1403.8727100629992		
covar ab1= -1503.7382507951434		
var c1= 76.3778557479906		
errort_min1 waspper= 0.0010337943529755607		
**************************************		
[-174.85260431 163.28018784 -28.11343588] [[442.64510073 -413.20994132 96.36730145]		
[-413.20994132 385.76804164 -89.97580012]		
[96.36730145 -89.97580012 20.98775453]]		
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Figure 2: Example of Re-Calculation 2

time mag mag_err		
0 0.373047 9.989208 0.003518	Input data	
1 0.373535 9.986668 0.003753		
2 0.387207 10.028049 0.003576		
3 0.387695 10.019831 0.003552		
4 0.401367 10.010567 0.003554		
5 0.401367 10.005163 0.003997		
**************************************		
-129.8054346400368 101.26598187766766 -9.725595108800313	2455897.3900 V	0.0007
T_min1 (waspper) = 0.3900683440508025		
var a1= 394.44524031038856		
var b1= 236.58781436097914		
covar ab1= -305.4672039302405		
var c1= 8.855925425617412		
errort_min1 waspper= 0.0007710503903867543		
**************************************		
[-129.80543464 101.26598188 -9.72559511] [[250.53983332 -194.02364267 37.53188832]		
[-194.02364267 150.27351206 -29.0722169]		
[37.53188832 -29.0722169 5.62501928]]		
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Figure 3: Example of Re-Calculation 3

References

Feigelson E. D., Babu G. J., 2012, Modern Statistical Methods for Astronomy, doi:10.48550/arXiv.1205.2064.

Smits D. P., Skelton P. L., 2019, , 67, 53